Design of Experiments within the Möbius Modeling Environment

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Abstract

Models of complex systems often contain model parameters for important rates, probabilities, and initial state values. By varying the parameter values, the system modeler can study the behavior of the system under a wide range of system and environmental assumptions. However, exhaustive exploration of the parameter space of a large model is computationally expensive. Design of experiments techniques provide information about the degree of sensitivity of output variables to various input parameters. Design of experiments makes it possible to find parameter values that optimize measured outputs of the system by running fewer experiments than required by less rigorous techniques. This paper describes the design of experiments techniques that have been integrated in the Möbius tool.

1 Introduction

Möbius is a discrete-event system modeling and analysis tool that has been widely used for the analysis of dependability, performability, and security properties of systems [1]. The Möbius framework supports multiple modeling formalisms and multiple solution techniques. The framework provides a common modeling environment that enables the creation of innovative new technologies as well as the comparison of new and existing technologies.

The Möbius framework provides support for model parameters that symbolically represent numerical values within the model specification. Before the model is solved, numerical values are assigned to the model parameters within experiments in a study module. By defining multiple experiments, the model can be solved for many different sets of parameter values without rewriting the model specification. Previously available study modules allowed users to create experiments by directly specifying parameters using sets and ranges. The existing studies are sufficient when the system modeler has a strong understanding of the relationship between the model parameters and the system, or when it is only necessary to vary one parameter at a time. However, those approaches can lead to an intractable number of experiments when many parameters need to be varied across a wide range in order to explore the interaction among them.

This paper describes the capabilities of a new study module in Möbius that incorporates design of experiments techniques that attempt to maximize the amount of information learned by each experiment and make it possible to reduce the number of experimental points required. Section 2 describes multiple techniques available in Möbius for designing experiments. Section 3 describes the process of validating and making predictions from an empirical model derived from the experimental data.

2 Experiment Design

Design of experiment strategies provide a systematic approach to defining experiments in order to reduce the overall number of experiments and thus reduce the total solution time, as well as to explore the interactions between factors. Factors in design of experiments correspond to model input parameters, and factor levels are the values those variables take. The strategies are included in a new design of experiment's study. When a new study is created, the user specifies the factors that should be varied, the factors that should have constant values, the reward measure to analyze as the response, and ,finally, the type of experimental design to create. Two categories of experiment designs are available in Möbius: two-level design strategies and response surfaces.

2.1 Two-level Design Strategies

There are two types of two-level design strategies available in Möbius: Plackett-Burman and two-level factorial design. Plackett-Burman (PB) designs are a special type of two-level design that allows for a large number of factors to be tested with a minimal number of experiments. PB designs are often useful for initial studies of large systems with many factors, when it is not known which factors have the largest influence on the response. Möbius supports up to 83 factors in PB designs. More details about the methodology are available in [2] and [3].

Factorial design strategies make it possible to incorporate the effect of interaction between factors. In a two-level factorial design, each factor is limited to two levels, the minimum and the maximum. Experiments are created for

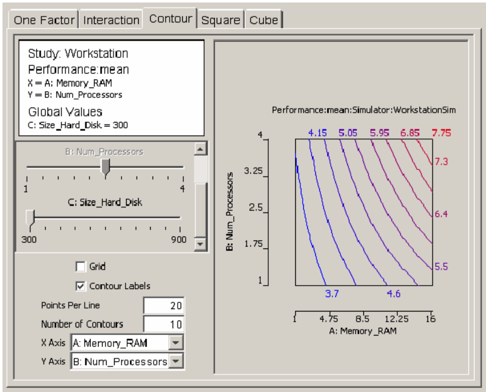

Figure 1. Contour response plot of performance of a computer workstation.

every combination of factor level, making the total number of experiments 2^K, where K is the number of factors.

When systems have a large number of factors, factorial designs can become too large to handle. Möbius supports fractional factorial designs, which alleviate the problem by only requiring that a fraction of the experiments be defined and solved. Although such designs reduce the amount of available information about the interaction among factors, the significant effects on the response can still be detected in fractional designs for the majority of models. Möbius provides 80 possible fractional factorial designs with up to 15 factors and 8,192 experiments.

2.2 Response Surface

Response surface designs are more sophisticated than factorial designs and are used to further refine the empirical model of the system, once the user has collected enough initial data using factorial designs. Response surfaces are typically constructed during the later stages of experimentation after the negligible factors have been eliminated. Response surface designs are employed when a higher-order (quadratic or cubic) polynomial model is necessary to accurately predict a response or when the user simply wants more detail on the response behavior in a specific region.

The Möbius design of experiments study supports the two most commonly used response surface design techniques: central composite (CCD) and Box Behnken designs. Each central composite design is based on an underlying factorial design and is often employed after the completion of a factorial analysis. Unlike CCD, the Box-Behnken design uses three levels for each factor. Box-Behnken is well-suited for design spaces in which regions under analysis are spherical and prediction of corner points is not important. See [2] for details on these techniques.

3 Analysis

An empirical model is created within the design of experiment study using the results computed by the Möbius solvers for the experiments created during the experimental design. The empirical model can be evaluated to predict the behavior of the system within some region of the design space. The accuracy of the optimal factor level depends on the accuracy of the empirical model that represents the real system. Multiple linear regression models are often used to estimate the response. The goal of the regression model is to accurately predict the true system response behavior.

To test the significance of regression, Möbius supports analysis of variance, or *ANOVA*. ANOVA makes it possible for the user to measure the significance of individual factors and the interactions between factors in the model. If the ANOVA shows that the regression model is significant, Möbius provides several diagnostic plots to determine whether the regression assumptions are correct. The diagnosis plots help the user determine whether the regression model provides an adequate approximation of the system being modeled. A measure of the predictive capability of the empirical model can also be determined from the plots. This predictive model can provide insight into response behavior for input variable levels not actually tested, and provide both confidence levels and confidence intervals for each predicted point in the design space. Figure 1 shows a typical contour response plot that can be generated to reveal the effect of factors on the measures of interest.

4 Conclusion

Design of experiments enables the modeler to use the results from a select number of simulations or numerical solutions to build an empirical model of the reward variable. Evaluation of the empirical model takes significantly less time than the evaluation of the true model. The empirical model (or regression models), when found to be satisfactory, can effectively replace simulation or numerical solution within specific areas of the design space.

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