A Kalman-based Coordination for Hierarchical State Estimation: Algorithm and Analysis

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***Abstract*—Hierarchical state estimation algorithms are usually employed in large-scale interconnected power systems, where state estimation usually involves very tedious communications and computations. This paper presents 1) a modified coordination technique that is based on Kalman Filtering, derived from hierarchical state estimation; and 2) a time complexity analysis and experimental implementation to compare central, distributed, and hierarchical state estimation algorithms in terms of computation power and communication bandwidth requirements. Analytical and experimental results on the IEEE 118-bus test bed show that the presented approach, i.e., Hierarchical Kalman Filtering (HKF), needs about 34% communication bandwidth and $O(\frac{1}{N^3})$ computation power in subsystems compared to central state estimation, while giving approximately the same level of estimation precision.**

I. Introduction

Nowadays, power networks are becoming more and more complicated and interconnected. Therefore, control centers feel the necessity of robust and scalable methods for power system state estimation that maintain performance suitably for large-scale systems. The state of a power system can be thought of as an independent set of n variables, i.e., bus voltage phasors, that completely describes its operating condition [10]. In practice, power system state is estimated using various sensor measurements that are classified as either *critical* or *redundant* [13]. Critical measurements are necessary to keep the system observable, while redundant measurements can be removed without affecting the system observability. Critical and redundant measurements can be identified in various ways, such as by checking the residual sensitivity matrix that represents the sensitivity of the measurement residuals to the measurement errors [11]. Generally, redundant sensors are placed to aid bad data detection algorithms; however, correlations among measurements, caused by redundant sensors, are employed to allow faster state estimators in power systems [10].

Recently, there has been an increasing interest in various types of state estimation algorithms [7]. The two most widely used power system state estimation algorithms are 1) the Weighted Least Squares (WLS), which minimizes the sum of squares of the weighted deviations of the estimated variables from the actual measurements [16]; and 2) the Kalman Filtering (KF) [9], which minimizes estimation error covariance. The main advantage of the KF over the WLS is in using system state information that improves the estimation precision.

The central KF state estimation algorithm can be used if there is no limit on communication channel and processing power. Complete measurements from all sensors are sent to a central state estimation unit, where the state of the whole system is estimated using these measurements. However, in physically spread-out large-scale systems, there are usually constraints on the amount of communications allowed among sensors, due to network congestion and security issues. Additionally, real-time Central State Estimation (CSE) in large-scale power systems with thousands of sensors is almost impossible due to processing power limitations.

Those communication and computation issues of the CSE motivate researchers to move toward Distributed State Estimation (DSE), for which state estimators are physically distributed across subsystems. Brice et al. [8] have investigated the feasibility of distributing computational load associated with the centralized state estimator to a set of smaller processors, thereby freeing the central processor for higher-level tasks and possibly providing more timely information on system state.

Patel and Girgis [3] propose a hierarchical state estimation algorithm for large-scale power systems for which centralized state estimation is unfeasible due to the communication cost and processing power limitations that results from using the WLS estimation algorithm. This paper presents: 1) a modified coordination technique for hierarchical state estimation that is based on Kalman filtering. Using the Kalman filter, the presented algorithm, i.e., the Hierarchical Kalman Filtering (HKF), exploits the state information and therefore increases estimation precision and convergence speed, even if the amplitude of value changes is high [12]. In the HKF, the power system is first decomposed into a set of interconnected observable subsystems that could be overlapping each other. After the system decomposition stage, local KF estimators estimate the local state of each subsystem using local critical and redundant measurements. There are some correlations among local estimates of subsystems due to tie-line measurements. Exploiting those correlations, the global estimator coordinates local estimates of subsystems to produce a system-wide system estimate. 2) analytical and experimental comparison among the HKF, central, and distributed state estimation in terms of communication bandwidth and computation power requirements.

This paper is organized as follows. Background literature is reviewed in Section II. In Section III, the modeling scheme is formulated. The Hierarchical Kalman Filter (HKF) and time complexity analysis are presented in Section IV. Finally, the HKF is implemented on an IEEE 118-bus test bed in Section V.

II. Related Work in Distributed and Hierarchical State Estimation

Recently, there has been an increasing interest in distributed power system state estimation. Korres et al. [2] proposed a distributed state estimator that is based on two-level state estimation, distributed observability analysis, and distributed bad data processing. Second-level estimation is accomplished using a reduced model.

Iwamoto et al. [17] developed a hierarchical computing scheme that includes two levels: the upper level, where the optimal tie-line bus voltages are evaluated, and the lower level, where the optimal states of each subsystem are determined. Their method makes use of an extension of a fast second-order load-flow technique that makes it possible to employ a fixed Jacobian matrix in the hierarchical algorithm. Zhao et al. [5], [4] examine the hierarchical state estimation on large-scale power networks using existing local estimators along with synchronized phasor measurements.

Lakshminarasimhan et al. [1], [6] propose to use hierarchical state estimation to coordinate state outputs of a wide area network. The authors also address issues of incoherency and and delay/absence of state output from one or more member areas. Patel et al. [3] implement hierarchical state estimation on a large real-world 1896-bus power system.

Aguado et al. [14] have employed decomposition techniques in distributed power system state estimation problems. They presented and compared two decomposition approaches based on Lagrangian Relaxation and Augmented Lagrangian algorithms. Additionally, they showed that using decomposition approaches in power system state estimation greatly reduces discrepancies among variables estimated at boundary buses by different control areas.

Ebrahimian et al. [15] presented an application of a parallel algorithm to power system state estimation. Using the Auxiliary Problem Principle, they developed a realistic and practical distributed power system state estimation algorithm that requires a minimum amount of modification to existing state estimators.

El-Keib et al. [18] presented a linear programming state estimation algorithm for large-scale interconnected power systems. Their algorithm uses the Dantzig-Wolfe Decomposition Algorithm (DWDA). By using a multiple-column strategy and several other modifications to the original algorithm, the authors improved its performance.

Lin [19] developed a robust and computationally efficient distributed state estimator to solve the Weighted Least Square (WLS) problem by using distributed computation for the power system. The proposed estimation algorithm offers four advantages over other estimators: 1) reduction of the time-skew problem; 2) freedom from power network topological errors; 3) easy identification of unobservable states; and 4) detection and identification of bad data.

There are two main incentives to move from central state estimation toward distributed and hierarchical approaches. First, local estimation in subsystems eliminates the necessity for communications between subsystems and the central estimation unit. Therefore, the communication bandwidth requirement is reduced. Second, local estimators, in hierarchical and distributed estimation, deal with much lower dimensional data compared to measurement vectors used in central estimation algorithms. Consequently, computational power requirement decreases with the use of hierarchical and distributed estimation techniques. Those two reasons have frequently been mentioned in the hierarchical and distributed estimation literature; however, there has been no coherent analytical and experimental comparison between hierarchical, distributed, and central state estimation. In this paper we propose a time complexity analysis and experimental evaluation of a modified coordination algorithm along with distributed and central estimation schemes.

III. Power System Modeling

Power system state estimation is based on the following linear stochastic difference model:

$$x_k = Ax_{k-1} + w_{k-1} \tag{1}$$

$$z_k = Hx_k + w_k \tag{2}$$

where $x \in \Re^n$ and $z \in \Re^m$ are state and measurement vectors, respectively. w is a random variable representing the process noise.

In large-scale power systems, where central state estimation is not feasible, distributed and hierarchical algorithms are usually employed. Suppose that the large-scale power system consists of N interconnected observable subsystems, $S_i \quad 1 \leq i \leq N$, with their corresponding local state and measurement vectors:

$$\begin{array}{ll} x_k^{(1)} = A^{(1)} x_{k-1}^{(1)} + w_{k-1}^{(1)} & z_k^{(1)} = H^{(1)} x_k^{(1)} + v_k^{(1)} \\ x_k^{(2)} = A^{(2)} x_{k-1}^{(2)} + w_{k-1}^{(2)} & z_k^{(2)} = H^{(2)} x_k^{(2)} + v_k^{(2)} \\ \vdots & \vdots \\ x_k^{(N)} = A^{(N)} x_{k-1}^{(N)} + w_{k-1}^{(N)} & z_k^{(N)} = H^{(N)} x_k^{(N)} + v_k^{(N)} \end{array} \tag{3}$$

where $x^{(i)} \in \Re^{n^{(i)}}$ and $z^{(i)} \in \Re^{m^{(i)}}$ are local state and measurement vectors of S_i, respectively ($n^{(i)} < m^{(i)}$).

The random variables $w^{(i)}$ and $v^{(i)}$ represent the process and measurement noise in S_i, respectively, which are assumed to be mutually independent:

$$p(w^{(i)}) \approx N(0, Q^{(i)}), \tag{4}$$

$$p(v^{(i)}) \approx N(0, R^{(i)}) \tag{5}$$

where Q and R are process and measurement noise covariances, respectively.

IV. Hierarchical Kalman Filtering

The Hierarchical Kalman Filtering (HKF) consists of three main steps: 1) Power System Decomposition (discussed in Section IV-A); 2) Local Kalman estimators (Section IV-B); and 3) the Global Kalman estimator (Section IV-C).

A. System Decomposition

The overall power system is decomposed into a set of interconnected subsystems, which may overlap each other, on a geographical basis. State vectors of these subsystems are then estimated independently by the HKF local estimators (Section IV-B). We call a system decomposition *valid* if it satisfies the following condition:

- Each subsystem must obey the *Observability Principle* [25]: its measurement Jacobian matrix must not have linearly dependent columns.

If the measurement model for the power system is given by:

$$z = h\left(x\right) + v \tag{6}$$

that means that measurement vector z is a nonlinear function $h\left(.\right)$ of the state vector $x \in \Re^n$. This nonlinear function could be locally linearized around the nominal operating point. The system is said to be observable if the matrix $H = \frac{\partial h}{\partial t}$, which relates measurements to states, is of rank n.

The above condition lets local estimators estimate local states in subsystems independently, without communicating with other subsystems. There could be more than one valid system decomposition scheme for a power system. Our validity criterion for system decomposition depends not only on network topology but also on sensor locations in the power system, since they both affect network observability.

Fig. 1 shows a sample system decomposition scheme for the IEEE 14-bus test bed. As shown in the figure, some subsystems overlap to satisfy local observability conditions.

B. Local Kalman Estimators

Once the power system is decomposed into subsystems, local Kalman estimator of each subsystem estimates its state using local measurements. The Kalman estimator minimizes the expected value of the square of the posterior state estimation error, $E\left[x_k - \hat{x}_k^-\right]$. We define $\hat{x}_k^- \in \Re^n$ to be the a priori state estimate at step *k* given knowledge of the process prior to step *k*, and $\hat{x}_k \in \Re^n$ to be the a posteriori state estimate at step *k* given measurement z_k. The KF has two distinct phases: *predict* and *update*. The *prediction* equations project forward the current state and error covariance estimates to obtain the a priori estimates for the next time step:

$$\hat{x}_k^- = A\hat{x}_{k-1} \tag{7}$$

$$P_k^- = AP_{k-1}A^T + Q \tag{8}$$

where P_k^- and P_k are a priori and a posteriori estimate error covariances, respectively:

$$P_k^- = E[(x_k - \hat{x}_k^-)(x_k - \hat{x}_k^-)^T] \tag{9}$$

$$P_k = E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] \tag{10}$$

The measurement *update* equations are responsible for the feedback needed to incorporate a new measurement into the a priori estimates to obtain an improved a posteriori estimate:

$$K_k = P_k^- H^T (HP_k^- H^T + R)^{-1} \tag{11}$$

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-) \tag{12}$$

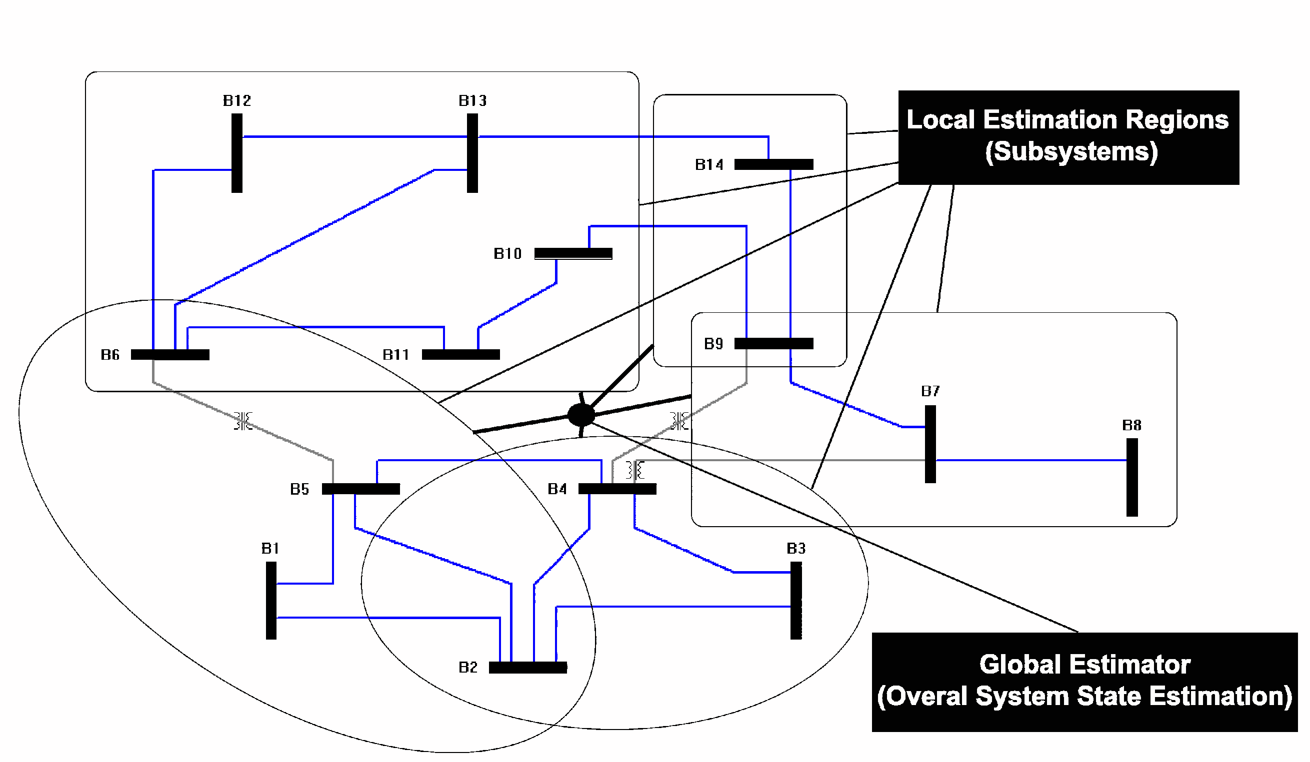

Fig. 1. A Sample System Decomposition for the IEEE 14-Bus Test Bed

$$P_k = (1 - K_k H) P_k^- \tag{13}$$

where $n \times m$ matrix K is the *gain* or *blending factor* that minimizes the a posteriori error covariance shown in Eq. 10.

Local measurements, in subsystems, are received from sensors periodically in each time step, i.e. $[t_k, t_{k+1})$. However, due to network problems, some of the measurements may not delivered to local estimators in predetermined time limit. In such situations, the last predicted values by the Kalman filter are used as new pseudo-measurements to local estimator.

C. Global State Estimation

After the system decomposition step, each subsystem are identified by its buses. Generally, buses in each subsystem are classified into two categories:

- Internal Bus: a bus whose neighboring buses are all within the same subsystem as itself.
- Boundary Bus: a bus that has some neighboring buses within its own subsystem, and some neighbors in other subsystems.

Subsystems are connected to each other by tie-lines between neighboring boundary buses. Once local estimates have been computed independently in the subsystems, tie-line measurements and local estimates are coordinated in a global estimator to yield a unified system-wide state estimate. The main idea of using the global estimator after the local estimation step is to exploit correlations among local estimates caused by tie-line measurements. The correlations are ignored in local estimations by local estimators and are due to tie-line measurements.

Fig. 2 illustrates data flow in the HKF estimation process. To get the system-wide state estimate, local estimates of all subsystems $\hat{x}_k^{(i)}$ and tie-line measurements $(z_{tlm}^{i,j})$ between any two neighboring subsystems S_i, S_j are sent to the global estimator. The global estimator considers these inputs as a new set of measurements. Therefore, the measurement vector for the global estimator would be:

$$z_k^T = \left[\left(\hat{x}_k^{(1)} \right)^T, \left(\hat{x}_k^{(2)} \right)^T, \ldots, \left(\hat{x}_k^{(N)} \right)^T, z_{tlm}^T \right]. \tag{14}$$

The global Kalman estimator, then, uses that measurement vector and power system modeling equation (Eq. 1) to obtain the optimal estimation of the whole system. The system-wide state is estimated in the global estimator using the same *prediction* and *update* equations as in the local Kalman estimators (Section IV-B).

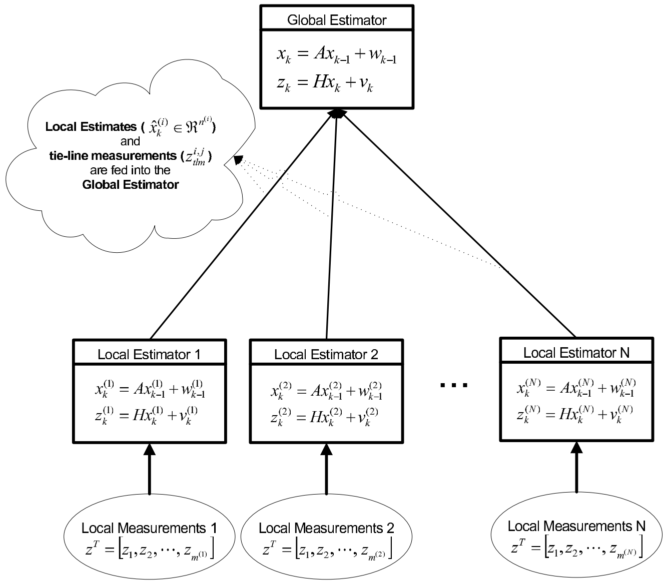

Fig. 2. The Hierarchical Kalman Filter (HKF) Data Flow

D. State Estimation Error Covariance

In addition to critical measurements, there are usually redundant measurements in power systems. These measurement redundancies, as an extra source of information, are used in state estimation algorithms to produce more precise estimates. The local estimators in the HKF state estimation algorithm estimate subsystems' states independently. Therefore, some sources of information are ignored. Here, we are investigating how estimation error covariance is affected by numbers of redundant measurements.

$$P_k = cov\left(x_k - \hat{x}_k\right) \tag{15}$$

Replacing $\hat{x}_k$ with Eq. 12:

$$P_k = cov\left(x_k - \left(\hat{x}_k^- + K_k\left(z_k - H\hat{x}_k^-\right)\right)\right) \tag{16}$$

and z_k with Eq. 2:

$$P_k = cov\left(x_k - \left(\hat{x}_k^- + K_k\left(Hx_k + v_k - H\hat{x}_k^-\right)\right)\right) \tag{17}$$

we get:

$$P_k = cov\left(\left(I - K_k H\right)\left(x_k - \hat{x}_k^-\right) - K_k v_k\right) \tag{18}$$

Since the measurement error v_k is uncorrelated with the other terms, this becomes:

$$P_k = cov\left(\left(I - K_k H\right)\left(x_k - \hat{x}_k^-\right)\right) + cov\left(K_k v_k\right) \tag{19}$$

By properties of vector covariance and use of the invariant on P_k^-, it becomes:

$$P_k = \left(I - K_k H\right) P_k^- \left(I - K_k H\right)^T + K_k R_k K_k^T \tag{20}$$

It is proved that if K_k is the optimal Kalman gain, yielding minimum mean-square error, Eq. 20 can be simplified further. The goal is to minimize the expected value of the square of the posterior state estimation error, i.e., $E\left[\mid x_k - \hat{x}_k \mid^2\right]$. That is equivalent to minimizing the trace of the posterior estimate covariance matrix P_k:

$$\frac{\partial tr\left(P_k\right)}{\partial K_k} = 2K_k\left(HP_k^- H^T + R_k\right) - 2\left(HP_k^-\right)^T \tag{21}$$

Solving Eq. 21 for K yields the optimal filter gain:

$$K_k = P_k^- H^T\left(HP_k^- H^T + R_k\right)^{-1} \tag{22}$$

Replacing Eq. 22 in Eq. 13 yields that the estimation error covariance iterate according to the following Algebraic Riccati Equation (ARE):

$$P_k = \left(I - P_k^- H^T\left(HP_k^- H^T + R\right)^{-1} H\right) P_k^- \tag{23}$$

The state estimator error covariance P_k asymptotically converges toward the steady state value P_∞ independent of the initial conditions. The two most common techniques for solving steady state of the Riccati equation are direct numerical methods, e.g., Newton method [20]-[21], and the Automatic Synthesis Program (ASP) matrix iterative procedure [22].

Rather than steady state analysis, we are more interested in focusing on how convergence speed is affected in the presence of redundant measurements. Here, we have solved Eq. 23 for a scalar constant signal without process noise and with m number of measurements. The measurement noise covariance matrix is $aI_{m\times m}$.

$$P_k = P_{k-1} - \frac{ma^{m-1}}{P_{k-1}^{m-1} d_m} \tag{24}$$

where d_m and its base value are:

$$d_m = \left(1 + \frac{a}{P_{k-1}}\right) d_{m-1} + \\ + \sum_{i=1}^{m-2}\left[\left(-1\right)^i\left(m-1\right)^i d_{m-1-i}\right] \tag{25}$$

$$d_1 = \left(1 + \frac{a}{P_{k-1}}\right) \tag{26}$$

Fig. 3 illustrates filter error covariance during state estimation iterations for the scalar constant signal. The solid line shows state estimation error covariance with

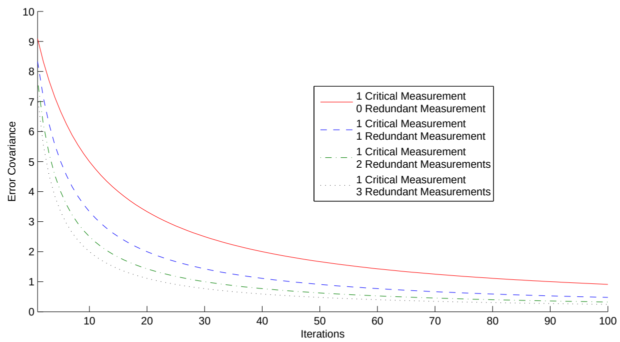

Fig. 3. How Measurement Redundancy Effects State Estimation Error Covariance

a critical measurement without measurement redundancy. Adding the first redundant measurement improves estimation precision significantly. However, the second and third redundant measurements do not substantially reduce the error covariance of the filter.

The CSE state estimation algorithms use all redundant measurements of the power system, whereas the HKF algorithm ignores some of the redundant measurements to reduce computational power and communication bandwidth requirements. Based on Fig. 3, if there are lots of redundant measurements in the system, the HKF estimation algorithm could be used, instead of the CSE, without affecting estimation error covariance significantly.

E. Complexity Analysis

In the CSE state estimation, the computation burden is mainly due to matrix multiplication. Recently, a couple of efficient matrix multiplication algorithms have been presented. Strassen [23] proposed a fast multiplication algorithm that takes $O\left(n^{log_2^7}\right)$, i.e., $O\left(n^{2.807}\right)$, for multiplication of two $n \times n$ matrices. The most efficient known algorithm is the Coppersmith-Winograd algorithm [24], which takes $O\left(n^{2.376}\right)$. For simplicity, in our analysis, we assume the standard matrix multiplication method is used, which multiplies two $n \times n$ matrices with $O\left(n^3\right)$ number of multiplications.

First, we calculate the complexity of the *prediction* equations. The state prediction equation (Eq. 7) involves the multiplication of $A_{n\times n}$ by the state vector $x_{n\times 1}$ that requires $O\left(n^2\right)$ computational cost. The error covariance prediction equation (Eq. 8) has two consequent matrix multiplications ($O\left(n^3\right)$) and a matrix addition ($O\left(n^2\right)$); therefore, error covariance prediction equation takes $O\left(n^3\right)$. Second, the complexity of *update* equations are taken into account. $H_{m\times n}P_{n\times n}H^T_{n\times m}$, in the Kalman gain computation (Eq.11), takes $O\left(mn^2\right)$. The $n \times n$ matrix inverse calculation needs $O\left(n^3\right)$. Consequently, the Kalman gain update computational cost is $O\left(mn^2+n^3\right)$. Similarly, the state estimate update (Eq.12) complexity is $O\left(mn+n^2\right)$. The last update equation, i.e., the error covariance update (Eq. 13) has a time complexity of $O\left(n^3\right)$. Consequently, the time complexities of the *prediction* and *update* equations are $O\left(n^3\right)$ and $O\left(mn^2+n^3\right)$, respectively.

Our analysis is based on a model for a large-scale power system that has been decomposed to N subsystems. Therefore, there are N local estimators with their corresponding state ($x^{(i)}_{n_i\times 1}$) and measurement vectors ($z^{(i)}_{m_i\times 1}$), with the following assumptions:

$$m_i = \frac{m}{N}, \qquad n_i = \frac{n}{N} \tag{27}$$

In the CSE estimation algorithm, in which all measurements are sent to a central estimator, the *prediction* and *update* equations have the following time complexity:

$$\begin{aligned} T\left(\textit{Prediction}\right): &\qquad O\left(n^3\right) \\ T\left(\textit{Update}\right): &\qquad O\left(mn^2+n^3\right) \end{aligned} \tag{28}$$

The time complexity of the HKF estimation scheme is analyzed separately for the local and global estimators. Local estimators in subsystems estimate using local state and measurement vectors:

$$\begin{aligned} T\left(\textit{Prediction}\right): &\qquad O\left(\left(\frac{n}{N}+\varepsilon\right)^3\right) \\ T\left(\textit{Update}\right): &\qquad O\left(m\left(\frac{n}{N}+\varepsilon\right)^2+\left(\frac{n}{N}+\varepsilon\right)^3\right) \end{aligned} \tag{29}$$

where ε is the dimension of tie-line measurements in each subsystem.

As explained in Section IV-C, all local estimates and tie-line measurements are fed into the global fusion center as its measurements. Therefore, dimension of measurement vector in the global estimator equals $N \times \left(\frac{n}{N}+\varepsilon\right)$.

$$\begin{aligned} T\left(\textit{Prediction}\right): &\qquad O\left(n^3\right) \\ T\left(\textit{Update}\right): &\qquad O\left(n^3+\left(n+N\varepsilon\right)n^2\right) \end{aligned} \tag{30}$$

In addition to processing power requirements, the HKF estimation reduces communication complexity compared to the CSE. Using Eq. 14, the dimension of global estimator measurement vector in the HKF algorithm is:

$$dim\left(z_{\textit{HKF GE}}\right) = \sum_{i=1}^{N} n^{(i)} \tag{31}$$

However, in the CSE estimation algorithm, the measurement vector would be:

$$z_k^T = \left[\left(z_k^{(1)}\right)^T, \left(z_k^{(2)}\right)^T, ..., \left(z_k^{(N)}\right)^T\right] \tag{32}$$

with its dimension equal to:

$$dim\left(z_{CE}\right) = \sum_{i=1}^{N} m^{(i)}. \tag{33}$$

Since $n^{(i)} < m^{(i)}$ (See Eq. 3):

$$dim\left(z_{HKF\ GE}\right) < dim\left(z_{CE}\right), \tag{34}$$

which shows the communication burden reduction in the HKF estimation algorithm compared to the central estimation scheme.

V. Experimental Evaluation

In this section, a case study is described to illustrate the concept of hierarchical state estimation in large-scale power systems. The IEEE 118-bus power system [26] is used for this purpose. *PowerWorld* simulator [27] is used to illustrate the IEEE 118-bus (See Fig. 4).

The global state vector x of this case study system is composed of voltage phasors of buses, i.e., 216 elements, while local state vector $x^{(i)}$ of each subsystem S_i consists of voltage phasors of the buses that belong to that subsystem. In the HKF estimation algorithm, local estimates are computed using local measurements. These measurements are usually provided by Phasor Measurement Units (PMUs).

PMUs provide two types of measurements: bus voltage phasors and branch current phasors. The number of channels used for measuring voltage and current phasors depends on the type of PMUs used. Here, we assume that each PMU has enough channels to record the bus voltage phasor at its associated bus and current phasors along all branches that are connected to this bus. Local measurement vectors include all internal bus PMU measurements and boundary bus PMU measurements, excluding tie-lines' current phasors. In the HKF estimation algorithm, the global estimator's input (measurements) vector from a particular subsystem consists of local state estimates and tie-line measurements of boundary bus PMUs on that subsystem.

The IEEE 118-bus test bed system is decomposed into 16 observable subsystems: $Si \quad 1 \leq i \leq 16$. Table I shows:

- Buses: System decomposition for the IEEE 118-bus system such that subsystems are observable.
- PMUs: Locations of Phasor Measurement Units (PMUs) in each subsystem.
- Local State Vectors (LSV): This column of the table shows the dimension of the state vector in each subsystem. The HKF and DSE state estimation algorithms use LSV vectors as operating state vectors in subsystems.
- Local Measurement Vectors (LMV): the dimension of the local measurement vector in each subsystem for local estimation is given in this column. In fact, these vectors are inputs to local estimators in the HKF and DSE algorithms.
- Measurement Vectors (MV): Numbers of measurements in each subsystem that are used in CSE state estimation algorithms.
- Output Vectors (OV): In the HKF estimation algorithm, after the local estimation step, the OV vector of each subsystem is sent to the global estimator. This column gives the dimension of the output vector for subsystems.

TABLE II
IEEE 118-Bus Test Bed Power System with the Same System Decomposition as in Table I and PMUs on Each Bus

Subsystem	LSV	LMV	MV	OV
S_1	26	90	98	34
S_2	30	98	120	52
S_3	10	26	38	22
S_4	24	72	90	42
S_5	6	18	24	12
S_6	10	34	50	26
S_7	12	36	48	24
S_8	18	58	70	30
S_9	12	32	54	34
S_{10}	10	34	46	22
S_{11}	2	2	10	10
S_{12}	24	80	100	44
S_{13}	6	14	22	14
S_{14}	26	90	108	44
S_{15}	14	46	56	24
S_{16}	6	14	18	10

In the HKF estimation algorithm, Local State Vectors (LSV) are estimated independently using Local Measurement Vectors (LMV). Then, the Output Vector (OV), including local state estimates and tie-line measurements, of each subsystem is sent to the global estimator. In the CSE algorithms, on the other hand, Measurement Vectors (MV) are sent to a central estimation unit. Compared to the CSE estimation algorithms, the HKF reduces the amount of data that each subsystem is supposed to send to the upper-level estimation unit. This reduction increases with the addition of more redundant measurements to the system. In Table I, the IEEE 118-bus system has been decomposed such that the subsystems are observable, but there are not a lot of redundant measurements. Table II shows the dimensions of the LSV, LMV, MV, and OV vectors for the IEEE 118-bus system with the same system decomposition as in Table I, but there is a PMU on every bus.

Fig. 5 illustrates the state estimation for the voltage magnitude of Bus 59 in the IEEE 118-bus case study system. Bus 59 is a boundary bus in subsystem S_{10} of our decomposed IEEE 118-bus system (Table I). The PMU on Bus 59 provides both local and tie-line measurements. The voltage phasor of Bus 59 and the current phasors of the 59-60, 59-63, and 59-61 transmission lines are the local

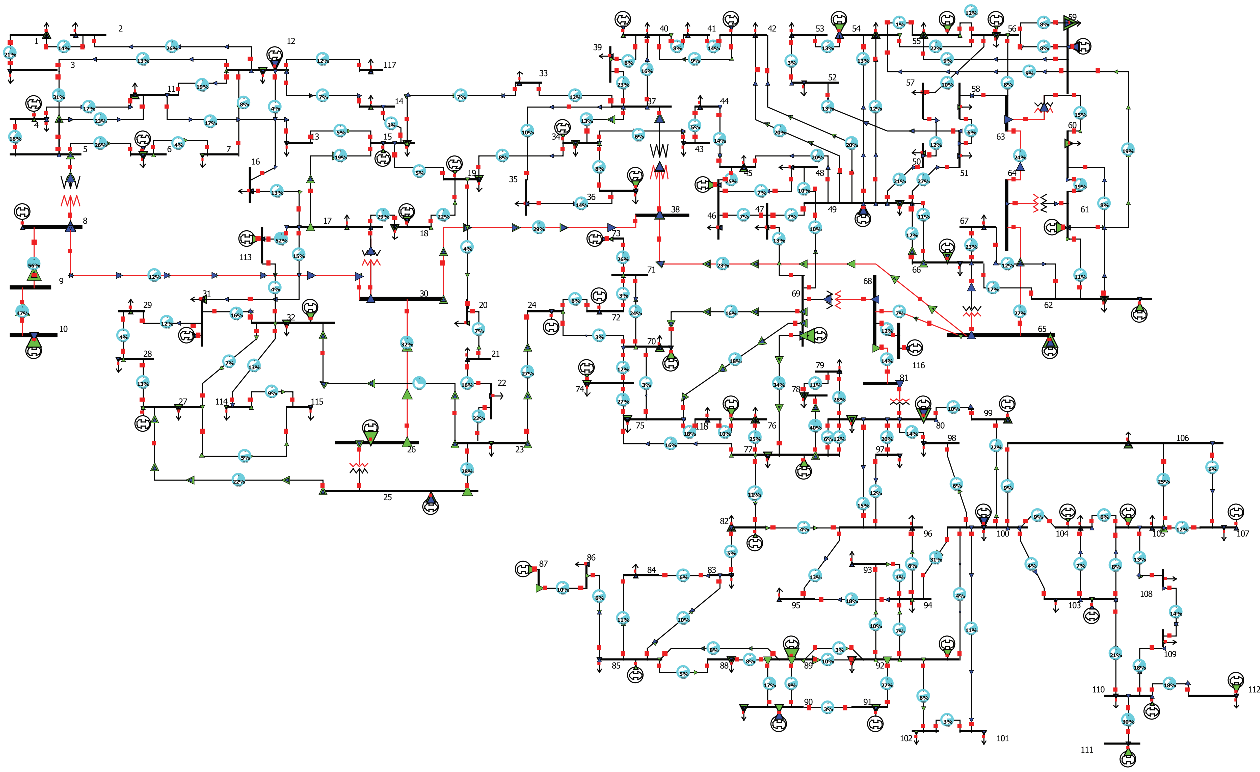

Fig. 4. IEEE 118-Bus Test Bed Power System

TABLE I
IEEE 118-BUS SYSTEM DECOMPOSITION, PMU LOCATIONS, AND DIMENSIONS OF LOCAL STATE VECTORS (LSV), LOCAL MEASUREMENT VECTORS (LMV), MEASUREMENT VECTORS (MV), AND OUTPUT VECTORS (OV).

Subsystem	**Buses**	**PMUs**	**LSV**	**LMV**	**MV**	**OV**
S_1	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 117	2, 5, 9, 11, 12	26	36	40	30
S_2	13, 14, 15, 16, 17, 18, 27, 28, 29, 30, 31, 32, 113, 114, 115	15, 17, 28, 114	30	34	38	34
S_3	22, 23, 24, 25, 26	23, 25	10	14	18	14
S_4	19, 20, 21, 33, 34, 35, 36, 37, 38, 39, 43, 44	21, 34, 37, 44	24	30	36	30
S_5	40, 41, 42	41	6	6	6	6
S_6	45, 46, 47, 48, 49	45, 49	10	14	28	24
S_7	70, 71, 72, 73, 74, 75	72, 73, 75	12	14	22	20
S_8	50, 51, 52, 53, 54, 55, 56, 57, 58	50, 52, 56	18	20	24	22
S_9	65, 66, 67, 68, 69, 116	66, 68	12	14	20	18
S_{10}	59, 60, 61, 63, 64	59, 61	10	16	24	18
S_{11}	62	62	2	2	10	10
S_{12}	76, 77, 78, 79, 80, 81, 82, 83, 84, 96, 97, 118	77, 80, 83, 118	24	32	44	36
S_{13}	85, 86, 87	86	6	6	6	6
S_{14}	88, 89, 90, 91, 92, 93, 94, 95, 98, 99, 100, 101, 102	89, 92, 94, 100	26	44	54	36
S_{15}	103, 104, 105, 106, 107, 108, 109	105, 109	14	16	18	16
S_{16}	110, 111, 112	110	6	6	10	10

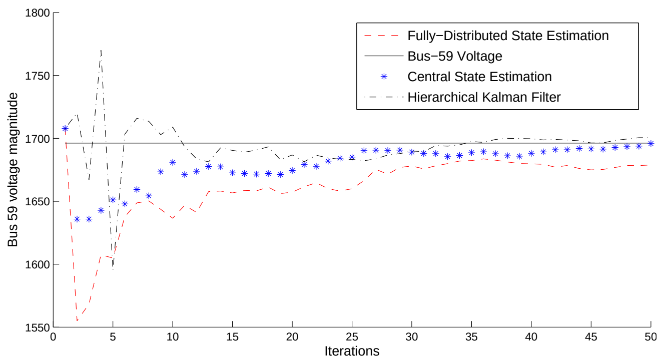

Fig. 5. Bus 59 Voltage Magnitude Estimation Process with CSE, DSE, and HKF State Estimation Algorithms

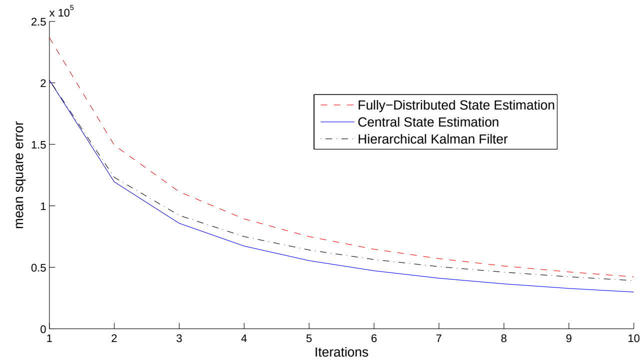

Fig. 6. Mean Square Error of the CSE, DSE, and HKF estimation algorithms

measurements. The tie-line measurements of the PMU on Bus 59 include the current phasors of the 59-55, 59-56, and 59-54 lines. Fig. 5 shows the Bus 59 voltage magnitude estimation process using three state estimation algorithms:

1) The Central State Estimation (CSE): As the CSE algorithm takes advantage of all measurements in the system, its estimates approach the actual voltage magnitude value faster than the other two estimation algorithms do. However, it has the most computation power and communication requirements (see Section IV-E, and Table I), which makes it usually infeasible to implement in large-scale power systems.
2) The fully Distributed State Estimation (DSE): In contrast to CSE algorithms, the DSE does not need to have a central estimation unit, so there is no communication cost between subsystems. This yields some information loss during subsystem state estimations, since when there are a large number of tie-line measurements, the measurements in each subsystem will have a larger impact on the state estimation solution at the neighboring subsystems. This impact is completely ignored in fully distributed state estimation.
3) The HKF state estimation: Using independent local estimators, the HKF decreases the computation and communication requirements of the CSE algorithms; however, the estimation precision is reduced because the global estimator no longer has access to all the actual measurements. Compared to the DSE algorithms, the HKF yields more fast and precise estimates of the system state, since it exploits interconnection measurements, i.e., tie-line measurements between neighboring subsystems.

Estimation precision is usually evaluated using the Mean Square Error (MSE) that is calculated using the following equation:

$$MSE\ (\hat{\theta}) = \frac{1}{n}\sum_{i=1}^{n}(\theta_i - \theta)^2 \tag{35}$$

where $\hat{\theta}$ is the estimator, θ is the estimated parameter, and θ_i are the samples.

Fig. 6 shows the MSE of the IEEE 118-bus system-wide steady state estimates during the estimation process using the CSE, DSE, and HKF algorithms. Although all three estimation algorithms eventually converge to the actual value, in first few iterations, the HKF behaves much like the CSE algorithms, but with lower computation and communication requirements. The MSE difference between the DSE algorithm and the other two approaches is substantial in the first iterations, since the DSE is missing measurement data from other subsystems. This MSE difference gradually converges to zero while the DSE estimators are receiving more measurement data during the estimation process. Having no communication at all between subsystems reduces DSE's estimation precision and speed. To reduce system state estimation MSE, the global estimator in the HKF exploits some of measurement data that are ignored in the DSE algorithm.

VI. Conclusions

A modified coordination technique for large-scale power system state estimation was presented that is based on Hierarchical Kalman Filtering (HKF). Additionally, analytical and experimental evaluations for hierarchical, central, and fully-distributed estimation schemes were given. Results on the IEEE 118-bus test bed power system show that the HKF significantly reduces the computation (up to 34%) and communication bandwidth ($O(\frac{1}{N^3})$) requirements compared to the central state estimation algorithms and give very similar convergence speed and estimation precision. Compared to the fully-distributed state estimation algorithms, the HKF has the same computation requirements for subsystems and yields more accurate solutions (up to 20% less MSE) and faster estimation convergence.

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