Equalizer-based performabilitiy modeling and control of DEDS

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Abstract

The focus of this contribution is on the performability modeling and analysis of the interdependence of system states.

State transition models successfully used to study the dynamic of stochastic discrete event dynamic systems (SDEDS) are stochastic PETRI nets (SPN). SDEDS examples are communication- manufacturing- and transportation systems.

This contribution shows how to use the SDEDS underlying stochastic process for performability modeling and analysis of the interdependence of the system states. The prediction of possible system states can be used for optimal state dependent supervisory control of the modeled system.

Using the state interdependencies it is possible to integrate a (stochastic) PETRI net model as a state observer of (S)DEDS. The methodology presented in this contribution is defined as *equalizer*.

Keywords: Communication- manufacturing- and traffic control systems, controlled MARKOV chains, stochastic discrete event dynamic systems (SDEDS), MARKOVIAN stochastic process, state dependent control, stochastic PETRI nets (SPN), supervisory control (alphabetically).

1 The modeling and analysis methodology

A state dependent control policy is derived for discrete event dynamic systems with countable state space; for which the constrained performability criteria arise naturally in the context of communication systems, manufacturing systems, and traffic control systems.

Dealing with practical applications, several authors have extended the basic PETRI net definitions to obtain a more powerful modeling tool. In particular, much work has been focused on the possibility of using PETRI nets for the control of discrete event dynamic systems and on representing time.

The introduction of functional specifications is essential in order to use PETRI net models for supervisory control of systems. This can be done by applying the transition invariants (T-invariants) and place invariants (S-Invariants) of the PETRI net model to generate the control rules for the modeled system. T-Invariants can be used to describe predefined walks through the state space of the modeled system. S-Invariants describe the invariance of the markings of the PETRI net model and can be used to simplify the analytically controller synthesis. If a conflict takes place within the generated control rules an appropriate strategy has to be found in order to solve the conflict. Here, T-Invariants and priorities can be used to describe the ordering of transition firings and thus to generate a conflict-free pre-defined walk through the state space of the modeled system.

In order to use PETRI nets as observer models and to implement a supervisory control of a deterministic dynamic system, the introduction of deterministic timing specifications is essential. It is advantageous that the implementation of time allowes the use of the PETRI net model for performance evaluation of the modeled system behaviour. Furthermore, if a stochastic system behaviour is considered (e.g. communication systems, manufacturing systems, and traffic control systems), the performance of the modeled system can be characterized using the stochastic processes, underlying the PETRI net model, of the modeled system.

The performance of the modeled deterministic or stochastic system can be characterized through the minimization of a long run avarage cost functional to satisfy pre-described bounds and subject to constraints on several other such functionals, e.g. useful for resource utilization.

To be able to depict the control methodeology, a reduced complexity with MARKOVian systems behaviour is considered in this contribution for the communication-, manufacturing-, or traffic control systems. Using the underlying MARKOVian stochastic process, the state dependent adaptive control of the parameters influencing the finite-state MARKOV chains leads to an optimal state dependent supervisory control of the stochastic discrete event dynamic system. The computation of the equilibrium solution of the MARKOVian stochastic process, underlying the discrete event dynamic system, is practically shown in this contribution.

The control policy, presented here, is a static one; future work deals with sensitivity- and pertubation analysis of the system parameters, used to control the system behaviour, resulting in a state dependent dynamic control policy. The use of the matrix exponential equation bridges the DEDS theory and the control theory. Thus, the contribution deeply bridges and applies stochastic PETRI net theory and adaptive control theory.

A collection of selected papers on PETRI net performance models can be found in JUANOLE *et al.* [1]. WESTPHAL [4, 3] deals with the use of PETRI net performance models for state dependent supervisory control of communication systems. WESTPHAL [2] deals with basic system parameters of a communications system; used to decide whether a SDEDS model has to be used to predict the system behaviour or no.

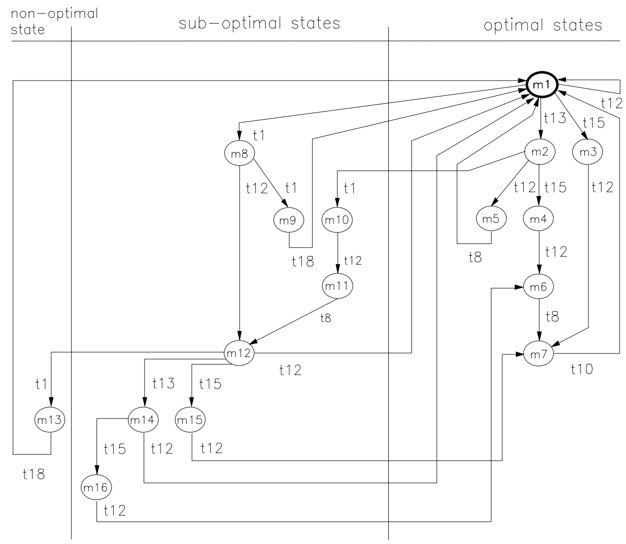

Figure 1: A tangible reachability graph.

2 Performability modeling and state dependent supervisory control

A tangible reachability graph, including all information that is required for the performability modeling of the modeled system, is shown in Figure 1, WESTPHAL [4]. From the tangible reachability graph the square transition rate matrix for equlibrium, $\mathbf{\Gamma} = [\gamma_{ij}]$, also called generator matrix, is defined. $\mathbf{\Gamma} = [\gamma_{ij}]$ is used to calculate the transition probability matrix $\mathbf{\Psi} = [\psi_{ij}]$. $\mathbf{\Psi} = [\psi_{ij}]$ and the initial probability density vector $\boldsymbol{\omega_{init}} = (\omega_{init\,1}, \ldots, \omega_{init\,N})$ are used to calculate the steady-state probability distribution vector $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_N)$ (N is the number of tangible states of the underlying stochastic process), Section 2.1.

For performability modeling and supervisory control, however, a vector notation for the mean number of transition firings $\overline{\boldsymbol{\gamma}} = (\overline{\gamma}_1, \ldots, \overline{\gamma}_n)$ (n is the number of timed transitions) can be defined, Section 2.2.

For state dependent performability modeling and state dependent supervisory control, however, a matrix notation for the state dependent (in terms of the MARKOV chain nomenclature, marking dependent in terms of the PETRI net nomenclature) mean number of transition firings $\overline{\mathbf{\Gamma}}^{\mathbf{T}} = [\overline{\boldsymbol{\gamma}}^{\mathbf{i}^{\mathbf{T}}}, \ldots, \overline{\boldsymbol{\gamma}}^{\mathbf{n}^{\mathbf{T}}}]$ can be defined, Section 2.3.

2.1 The steady-state probability distribution vector ω

Since the considered stochastic PETRI net model is defined to generate an ergodic continous time MARKOV chain it is possible to compute the steady-state (i.e. $\lim_{t\to\infty} \frac{d}{dt}\, \omega_i(t) = 0$) probability distribution vector $\boldsymbol{\omega}$, by solving eqn 1,

$$\boldsymbol{\omega}\ \mathbf{\Gamma} = \mathbf{0} \qquad \forall\ \boldsymbol{\omega}\ :\ \sum_{i=1}^{|\mathbf{s}(\mathbf{\Pi_{tim}})|} \omega_i = 1, \tag{1}$$

where $\mathbf{\Gamma} = [\gamma_{ij}]$ is the square transition rate matrix for equlibrium (i.e. $\sum_{j=1}^{|\mathbf{s}(\mathbf{\Pi}_{tim})|} \psi_{kj} = 0,\ k \in \{1, \ldots, |\mathbf{s}(\mathbf{\Pi}_{tim})|\}$), also called generator matrix.

The elements γ_{ij} denote the rate of transition of the MARKOV chain from state i to state j. $|\mathbf{s}(\mathbf{\Pi_{tim}})|$ is the number of tangible states of the underlying stochastic process, it is defined to N; this in order to get an easier nomenclature, i.e. $N := |\mathbf{s}(\mathbf{\Pi_{tim}})|$. $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_N)$ is the stationary probability density vector, i.e. ω_i is the probability of state i, if the system is in stochastical equlibrium.

However, with the square matrix $\mathbf{\Psi} = [\psi_{ij}]$, eqn 2,

$$\mathbf{\Psi} = \mathbf{\Gamma}\ \theta + \mathbf{I} \tag{2}$$

eqn 1 may also be written as eqn 3,

$$\boldsymbol{\omega}\ \mathbf{\Psi} = \boldsymbol{\omega} \qquad \forall\ \boldsymbol{\omega}\ :\ \sum_{i=1}^{N} \omega_i = 1 \tag{3}$$

With the scalar θ (see eqn 2) we get $\mathbf{\Psi}$ as a stochastic matrix (i.e. $\sum_{j=1}^{N} \psi_{|kj|} = 1,\ k \in \{1, \ldots, N\}$). The steady-state probability distribution $\boldsymbol{\omega}$ is iteratively calculated and derived by using eqn 4,

$$\boldsymbol{\omega}^{\mathbf{j+1}} = \boldsymbol{\omega}^{\mathbf{j}}(\ \mathbf{\Gamma}\ \theta + \mathbf{I}\ ). \tag{4}$$

The matrix $\mathbf{\Gamma}$ is, as mentioned before, called the transition rate matrix for equlibrium, or the infinitesimal generator of the MARKOV chain, while the matrix $\mathbf{\Psi}$ is called the transition probability matrix.

With $|\boldsymbol{\omega}^{\boldsymbol{j}}$ - $\boldsymbol{\omega}^{\boldsymbol{j-1}}| \leq$ **eps** (**eps**ilon, pre-defined). the iteration, eqn 4, is stopped after $j - times$.

Note 1: Scince the control policy, presented here, is a static one, the matrix $\boldsymbol{\Psi}$ is *not* changed by the iteration process and the steady-state probability distribution depends *not* on the initial probability distribution $\boldsymbol{\omega_{init}} = (\omega_{init\,1}, \ldots, \omega_{init\,N})$.

Note 2: The scalar θ must be choosen in an optimal way depending on $\boldsymbol{\Gamma}$. θ influences the convergence of the algorithm and thus the number of iterations (for a defined **eps**), that are needed to calculate the steady-state probability distribution. Furthermore, adapting θ, $\boldsymbol{\Psi}$, i.e. $\sum_{j=1}^{N} \psi_{|kj|} = 1,\ k \in \{1, \ldots, N\}$, must be verified. Thus, θ affects the robustness of the *equalizer*.

2.2 The mean number of transition firings, $\overline{\gamma_i}$ of t_i

For performance modeling and supervisory control, however, a vector notation $\overline{\boldsymbol{\gamma}} = (\overline{\gamma}_1, \ldots, \overline{\gamma}_n)$ for the mean number of firings $\overline{\gamma}_i$ of transition t_i can be defined. With the steady-state probability distribution vector $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_N)$, for example, we get eqn 5

$$\overline{\gamma}_i = \sum_{j=1}^{N} \omega_j \ \frac{\gamma_{i_j}}{-\Sigma_i} \quad \text{with} \quad \Sigma_i = -\sum_{j=1}^{n} \gamma_{i_j}, \quad \forall \ j \neq i \ , \tag{5}$$

where γ_{i_j} is the firing rate of t_i after the system was in state j, and $-\Sigma_i$ is the sum of the firing rates of the transition t_i, leaving the particular state j; keeping this in mind, a vector notation for the mean number of firings can be defined, eqn 6,

$$\overline{\boldsymbol{\gamma}} = (\overline{\gamma}_1, \ldots, \overline{\gamma}_n). \tag{6}$$

2.3 State dependent (marking dependent) mean number of transition firings, $\overline{\gamma}_{i_j}$ of t_i

For performance modeling and state dependent supervisory control, however, a matrix notation for the state dependent (marking dependent) mean number of firings $\overline{\gamma}_{i_j}$ can be defined, eqn 7,

$$\overline{\boldsymbol{\Gamma}}^{\mathbf{T}} = [\overline{\boldsymbol{\gamma}}^{\mathbf{i}^{\mathrm{T}}}, \ldots, \overline{\boldsymbol{\gamma}}^{\mathbf{n}^{\mathrm{T}}}] \tag{7}$$

with $\overline{\boldsymbol{\gamma}}^i = (\overline{\gamma}_{i_j}, \ldots, \overline{\gamma}_{i_n})$ and $\overline{\gamma}_{i_j} = \omega_j \ \frac{\gamma_{i_j}}{-\Sigma_i}$.

3 Some performability results

The performability estimates can be used for supervisory control of the behaviour of the modeled SDEDS. If lumpability at the optimal states is stressed, Figure 1, the system performs optimal with no direct reliability problems (non-optimal state), however, there is a chance that the system will reach the non-optimal state via the sub-optimal states. If lumpability at the sub-optimal states is stressed, the system performs sub-optimal with a chance of optimal performance and a chance of reliability problems. If lumpability at the non-optimal state is stressed, the system has a reliability problem and there is only a chance to performs optimal after solving the problem. Stressing lumpability at sub-optimal and non-optimal system states may be used to predict system faults and ease maintenance and repair tasks.

The derived performability estimates can be used for optimal state dependent supervisory control of SDEDS and are applied for a supervisory control of a real system, WESTPHAL [4, 3].

3.1 Example: The steady-state probability distribution vector $\boldsymbol{\omega}$

The square transition rate matrix for equlibrium, $\boldsymbol{\Gamma} = [\gamma_{ij}]$, defines the transition firing rates. For example, the transition firing rate $\gamma\, i_j$ is the firing rate of transition i, firing after the system was in state j. With the scalar θ, we get the transition probability matrix $\boldsymbol{\Psi} = [\psi_{ij}]$, that is used to calculate $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_N)$ by an iteration process, that is stopped after $j - times$ if $|\boldsymbol{\omega}^{\boldsymbol{j}}$ - $\boldsymbol{\omega}^{\boldsymbol{j-1}}| \leq$ **eps**.

For the example shown in Figure 1 the following transition firing rates for $\boldsymbol{\Gamma} = [\gamma_{ij}]$ for equilibrium are defined, WESTPHAL [4]: $\gamma 1_1 = \gamma 1_2 = \gamma 1_8 = \gamma 1_{12} = \gamma 18_9 = \gamma 18_{13} = 5$; $\gamma 10_7 = \gamma 12_1 = \gamma 12_2 = \gamma 12_3 = \gamma 12_4 = \gamma 12_8 = \gamma 12_{10} = \gamma 12_{12} = \gamma 12_{14} = \gamma 12_{15} = \gamma 12_{16} = 10$; $\gamma 13_1 = \gamma 15_1 = \gamma 15_2 = \gamma 13_{12} = \gamma 15_{12} = \gamma 15_{14} = 20$; $\gamma 8_5 = \gamma 8_6 = \gamma 8_{11} = 30$.

Furtermore, $\theta = 0.018$ (because of convergence), i.e.

$$\boldsymbol{\Psi} = \begin{cases} \sum_{j=1}^{N} \psi_{|kj|} = 1,\ k \in \{1, \ldots, N\} & : \ \theta > 0 \\ \sum_{j=1}^{N} \psi_{|kj|} = 1,\ k \in \{1, \ldots, N\} & : \ \theta \leq 0.0\overline{22}, \end{cases}$$

and $eps = 0.1 * 10^{-6}$, and $\boldsymbol{\omega_{init}} = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ have to be defined.

Figure 2 shows the transient states, derived by iterating eqn 4 $j - times$, i.e. $j = 1, 2, 5, 10, 20, 50, 100$, respectively, to reach the stationary system state at iteration $j = 101$. The steady-state probability distribution vector $\boldsymbol{\omega}$ is calculated to $\boldsymbol{\omega} = (0.0945, 0.0540, 0.1891, 0.1081, 0.0180, 0.0407, 0.3326, 0.0315, 0.0315, 0.0270, 0.0090, 0.0106, 0.0106, 0.0071, 0.0213, 0.0142)$.

3.2 Example: The mean number of transition firings, $\overline{\gamma_i}$ of t_i

With the steady-state probability distribution $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_N)$ we can find the mean number of firings of the transitions in unit time, used as a control policy, e.g. to utilize resources.

The mean number of firings $\overline{\gamma_i}$ of t_i in unit time is calculated using eqn 5. A vector notation for the mean number of firings $\overline{\boldsymbol{\gamma}}$ is defined in eqn 6. For the calculation of the mean number of firings the components of the steady-state probability distribution vector $\boldsymbol{\omega}^{101}$, eqn 4, are used. More information on the use of $\overline{\boldsymbol{\gamma}}$ for supervisory control of SDEDS can be found

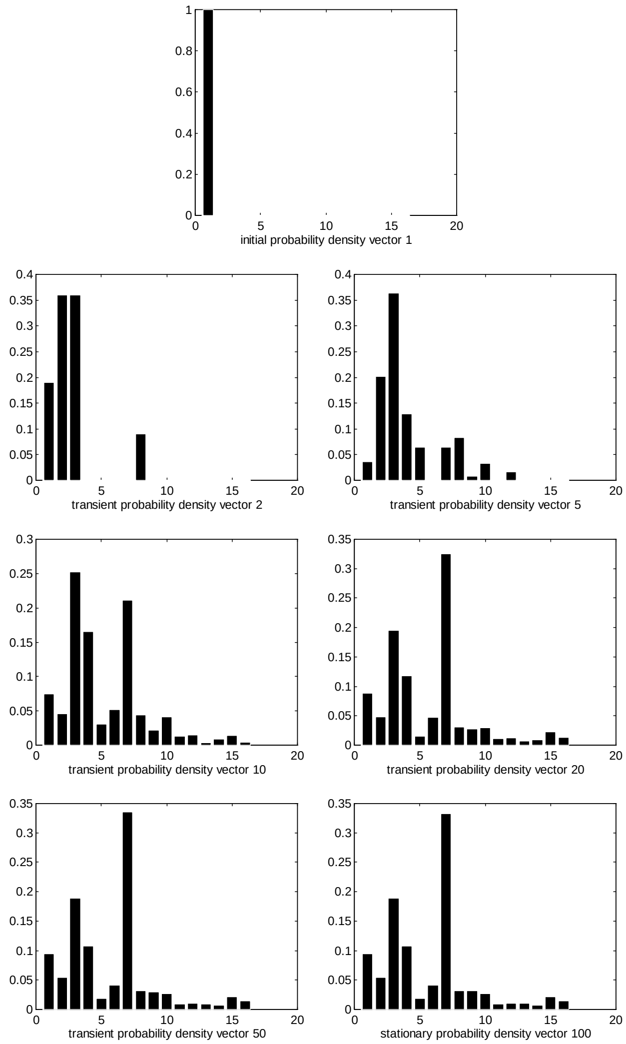

Figure 2: The state probability density vectors $\boldsymbol{\omega}$ for $j - time$ iterations, i.e. $j = 1, 2, 5, 10, 20, 50, 100$.

in WESTPHAL [3, 5]. The calculated mean number of firings, $\overline{\gamma} = (\overline{\gamma}_1, \ldots, \overline{\gamma}_N)$, are as follows: $\overline{\gamma}_N$ = (0.027786, 0.067761, 0.332621, 0.417572, 0.038250, 0.073853, 0.042156).

3.3 Example: State dependent (marking dependent) mean number of transition firings, $\overline{\gamma}_{i_j}$ of t_i

For performability modeling and state dependent supervisory control, however, a matrix notation for the state dependent (marking dependent) mean number of firings $\overline{\gamma}_{i_j}$ is defined in eqn 7. For the SDEDS underlying stochastic process the state dependent mean numbers of firings $\overline{\gamma}_{i_j}$, denoted in percent %, are derived for transition t_1 as follows: $\overline{\boldsymbol{\gamma}}_{\mathbf{1}}$ = (0.859, 0.772, 0, 0, 0, 0, 0, 1.051, 0, 0, 0, 0.097, 0, 0, 0, 0).

Using the equilibrium, steady-state, and marking dependent probability distribution of the transition firings, eqn 7, performability estimates of the modelled system can easily be obtained. The derived performability estimates can be used for state dependent supervisory control of the behaviour of the SDEDS and if lumpability at the optimal system states is stressed, the derived performance estimates can be used for optimal supervisory control of the systems behaviour. More information on the use of $\overline{\gamma}_{i_j}$ for prediction, scheduling, and optimal supervisory control of SDEDS can be ordered from the author.

4 Conclusion

The stochastic Petri net concept provides a systematic effectual development for the analysis and state dependent control of the dynamic behaviour of SDEDS. The Petri net concept is further applicable to complex SDEDS in which distributed control strategies need centralized supervisory control. Since this methodology is systematic, various levels of hierarchies and thus complexities can easily be coordinated.

The contribution should be viewed as a modeling, analysis, and state dependent supervisory control methodology, defined as *equalizer*, based on fundamental principles for state dependent control of SDEDS. The visualization of the state dependent probabilities serves to depict the non-linear parameter dependencies while adapting the system parameters.

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